Generalized Third-Order Law in MHD Turbulence with Distinct Dissipation Mechanisms

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Introduction

In this study, we investigate the energy cascade in Alfvénic solar wind turbulence under non-ideal MHD conditions, where viscosity (ν) and resistivity (μ) differ and act at separate scales [1]. Recent observations suggest that viscous effects dominate at larger scales than magnetic dissipation. Adopting a phenomenological model with ν ≠ μ , we study how this imbalance impacts the energy transfer, focusing on the third-order Yaglom law reformulated using Elsässer variables. This relation directly measures the cascade rate and reveals asymmetries in kinetic and magnetic dissipation. Through theoretical analysis and ongoing simulations, our goal is to quantify these effects and improve the interpretation of in-situ measurements in the solar wind and magnetosheath.

Numerical Model

The starting points are the incompressible MHD equations, written in terms of the Elsässer variables [2]

$$z^{\pm} = \vec{v} \pm \frac{\vec{B}}{\sqrt{4\pi\rho}}$$

which represents Alfvénic fluctuations propagating along the background magnetic field in opposite directions.

We rewrite the MHD equation [3] at two points, in order to express it in terms of increments in space. And we take the average $\Delta\vec{z}^{\pm} = \vec{z}'^{\pm} - \vec{z}^{\pm}$.

$$\underbrace{\frac{\partial}{\partial t} < |\Delta\vec{z}^{\pm}|^2 >}_{} + \underbrace{\frac{\partial}{\partial r} < \Delta\vec{z}^{\mp}|\Delta\vec{z}^{\pm}|^2 >}_{} = \frac{\partial^2}{\partial r^2}\underbrace{\left[(\nu+\mu) < |\Delta\vec{z}^{\pm}|^2 > + (\nu-\mu) < \Delta\vec{z}^{+}\cdot\Delta\vec{z}^{-} >\right]}_{} \underbrace{- 2\epsilon}_{}$$

- 🔵 Trasport Term
- 🟣 Cascade Term
- 🟢 Dissipation Term
- 🔴 Energy Dissipation Rate $\epsilon = (\nu+\mu) < |\vec{\nabla}\vec{z}^{\pm}|^2 > + (\nu-\mu) < (\vec{\nabla}\times\vec{z}^{\mp})\cdot(\vec{\nabla}\times\vec{z}^{\pm}) >$

The simulations are performed in a periodic domain without external forcing.

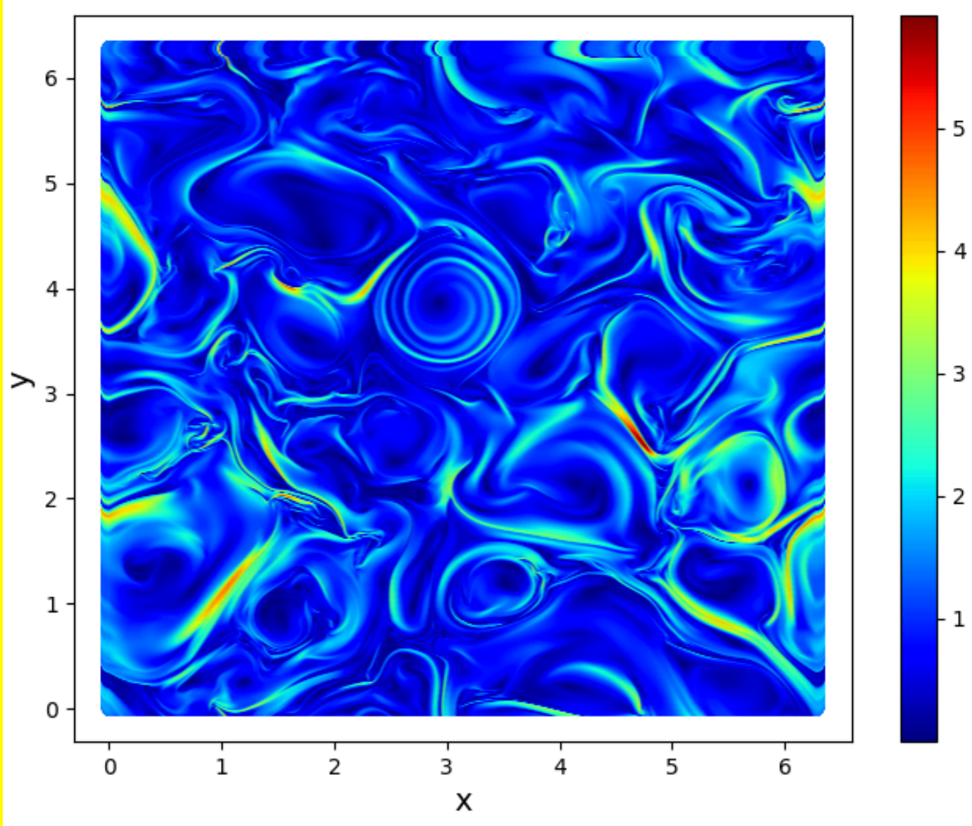
- Grid size: **4096 x 4096**
- Time step: **Δt= 0.00008**

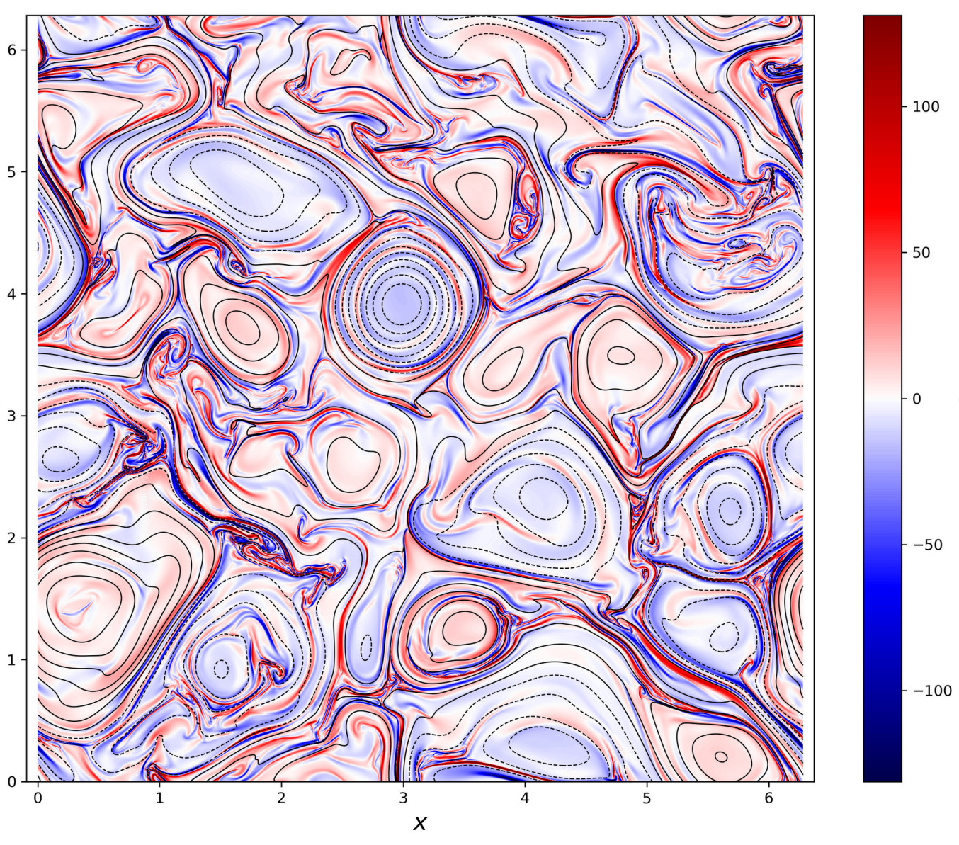
a) Dissipation coefficients: $\nu = \mu = 1\times10^{-4}$ $P_m=1$

b) Dissipation coefficients: $\nu = 1\times10^{-3}$, $\mu = 1\times10^{-4}$ $P_m=10$

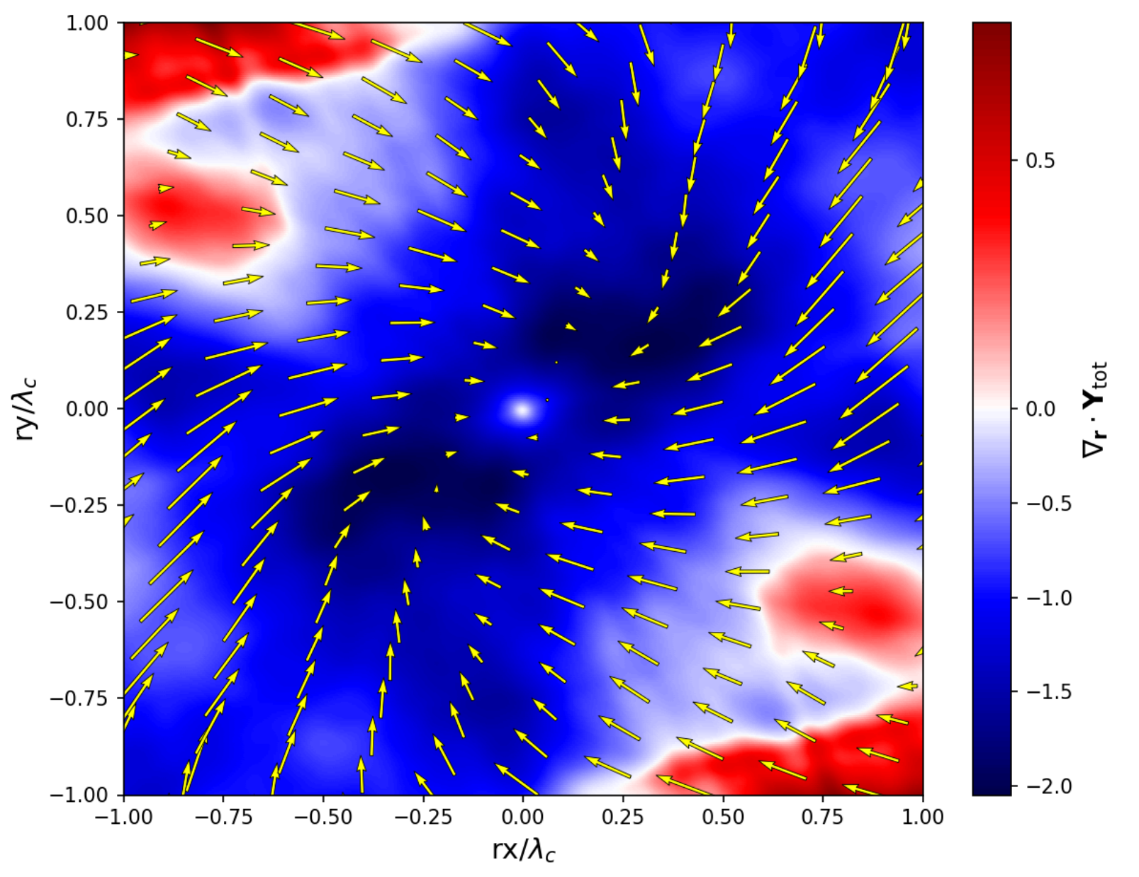
c) Dissipation coefficients: $\nu = 1\times10^{-4}$, $\mu = 1\times10^{-3}$ $P_m=0.1$

Results

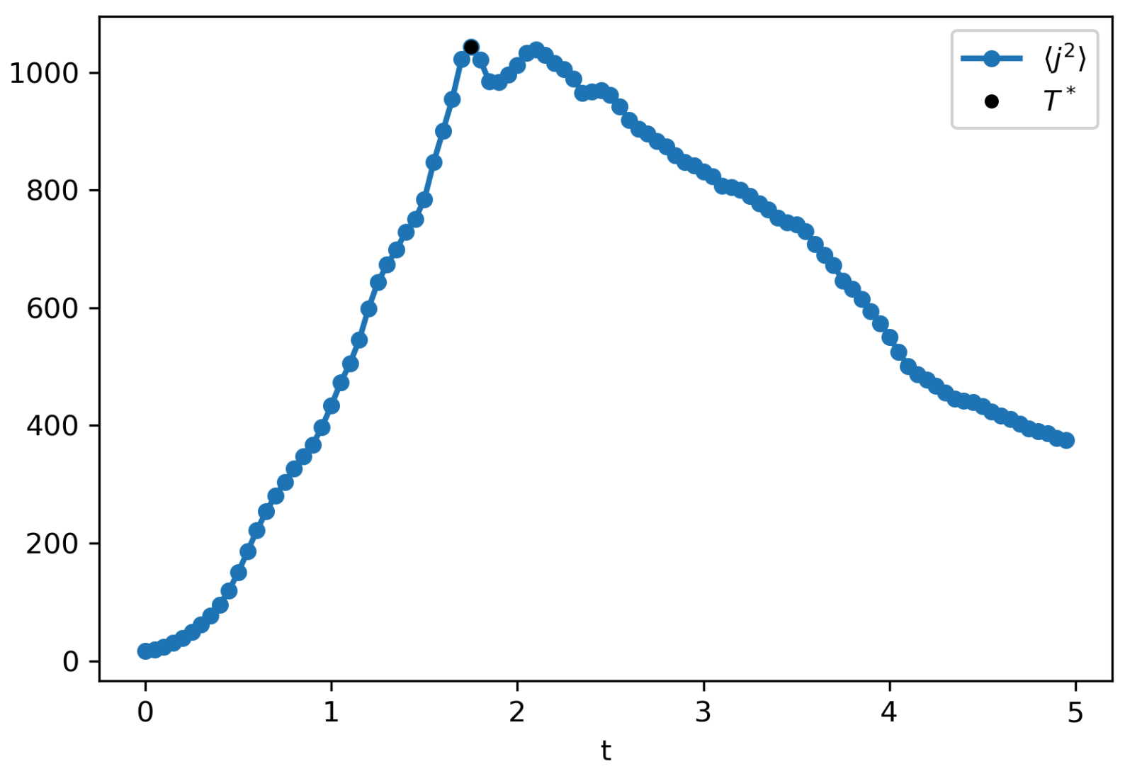




1. Spatial distribution of the Elsässer variable z, consistent with statistically homogeneous turbulence.
2. Spatial distribution of J_z with contours of A_z, representing in-plane magnetic field lines and highlighting coherent current structures in MHD turbulence.



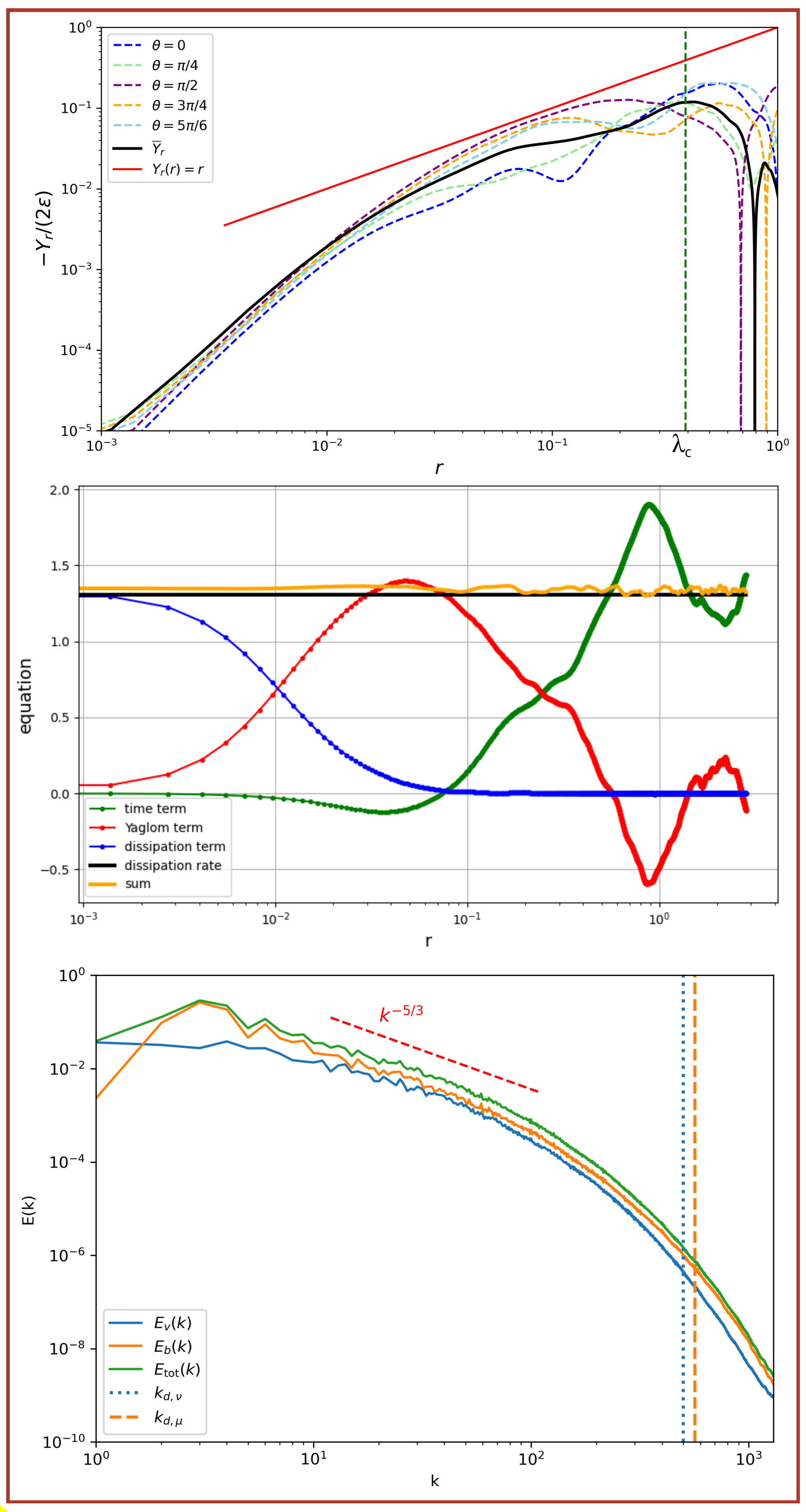
Divergence of the total Yaglom flux and the associated vector field, illustrating the intensity, direction, and anisotropy of energy transfer across scales in MHD turbulence.



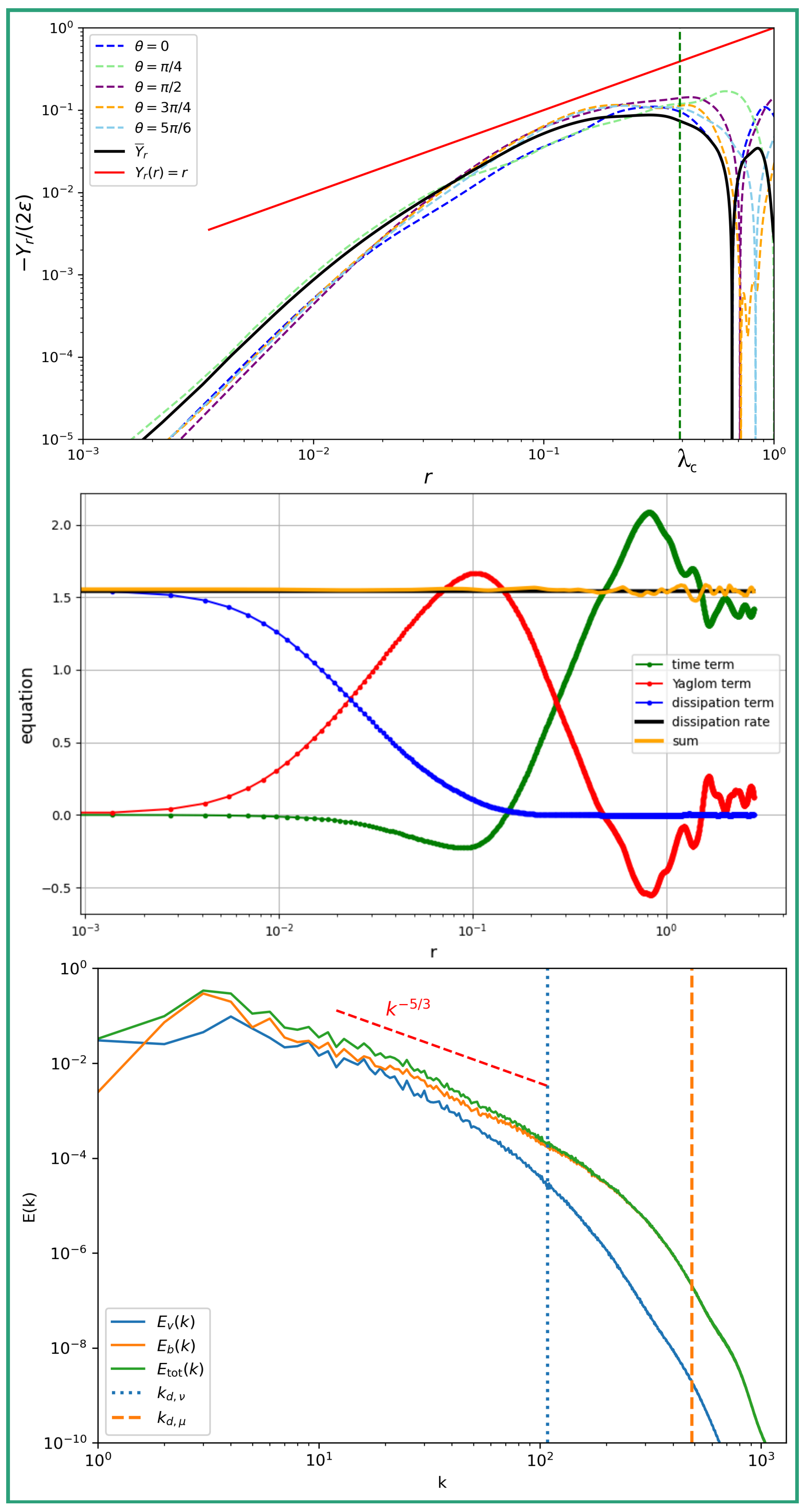
The time evolution of the current density. The peak, indicated by the black dot at T*, corresponds to the point of maximum turbulence intensity.

Parametric study

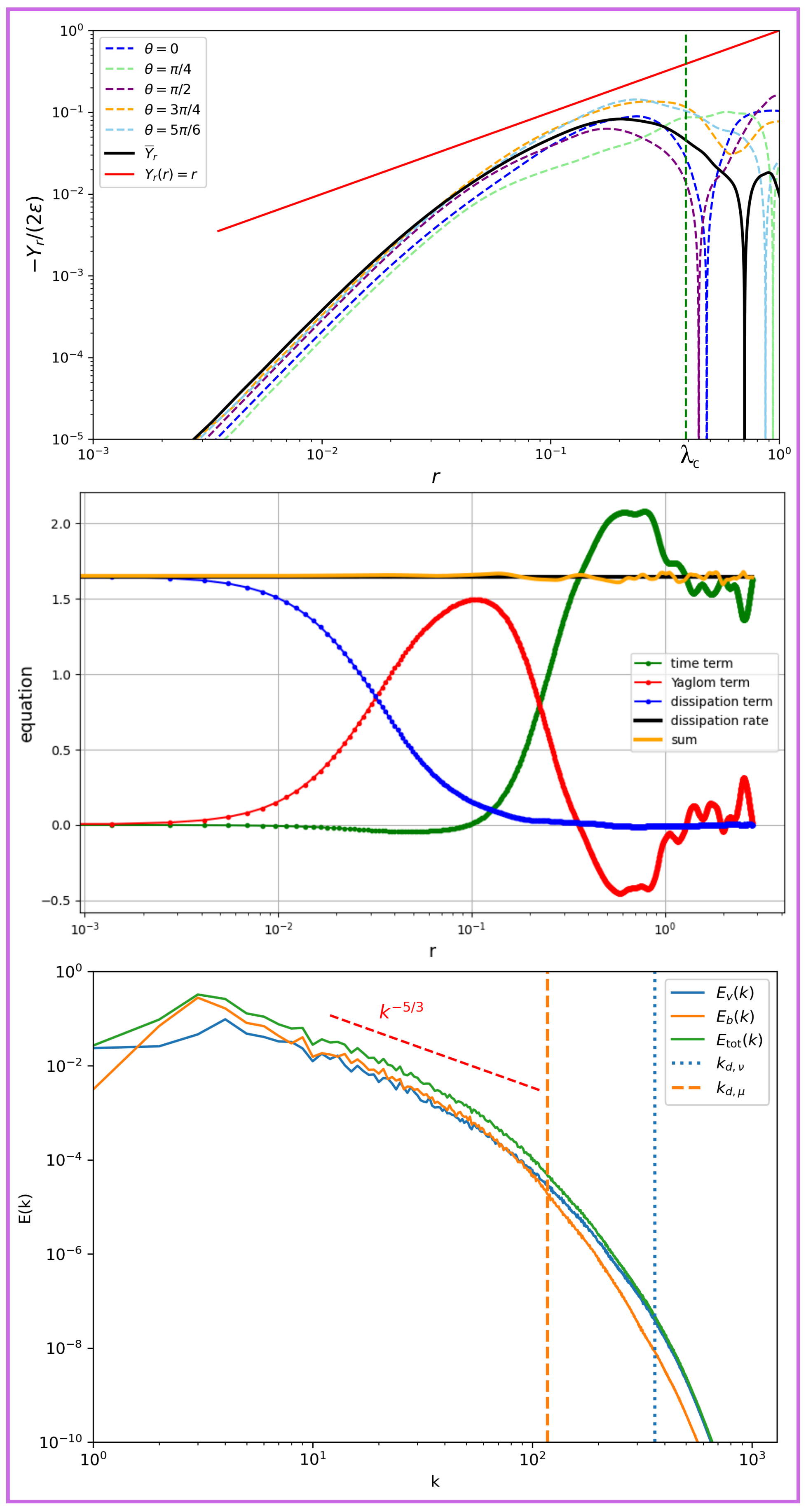
(a)



(b)



(c)



Radial component of the Yaglom flux $Y_r(r)$, nomalized via −2ε, at several angles (lines). The polar average (along θ), is reported with black line.

Terms of the equation, showing the three main contributions to the energy budget. The black line indicates the exact dissipation rate, and the orange curve shows the sum of the three terms.

Total energy spectrum E(k) as a function of the wavenumber k, exhibiting Kolmogorov scaling typical of inertial-range turbulence. Vertical lines indicate the viscous and magnetic dissipation scales.

References

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- Carbone V.,et al., 2009, On the turbulent energy cascade in anisotropic magnetohydrodynamic turbulence. Europhysics Letters, 88(2), 25001.
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Conclusions

- Following the Yaglom approach, we analytically derive a new third-order law describing the energy cascade in MHD turbulence in different physical scenarios involving different Prandl number.
- We validate the new law through direct MHD simulations for varying values of ν e μ, investigating the scale-by-scale energy transfer for the different cases.
- The results clearly show that the Prandl number influences the dynamics of the cascade.